**Business Analytics: review problems. Solutions provided at the end.**

**Problem 1:**

A company produces two products, A and B. Each unit of A requires 1, 3 and 2 kilograms of wood, plastic and steel respectively and each unit of B requires 3, 4 and 1 kilograms of wood, plastic and steel respectively. A maximum of 240, 360, and 180 kilograms of wood, plastic and steel available. The profit per unit of A and B is $4.00 and $6.00 respectively. The objective is to maximize total profits. Formulate this as a linear programming model.

**Problem 2:**

A manufacturer produces desks and chairs. Each desk uses 5 units of wood and each chair uses 3 units of wood. Total wood available for a month is 2200 units. Desk production requires 3 hours of labor and a chair needs 1.4 hours. Total number of hours available per month: 1150. Each desk contributes 40 dollars to profit and each chair contributes $22. Marketing requires that at least 3 chairs be produced for each desk produced. The objective is to maximize total profits. Note: you can ignore the integer requirements. Formulate this as a linear programming model

**Problem 3:**

A resort hotel is being built in a wooded area. Four locations (“nodes”) are to be connected with paths. All locations must be connected. Building paths is costly, so the objective is to minimize the total distance of building all the paths. Formulate this as a linear program.

|  |  |  |
| --- | --- | --- |
| Table 1 | | |
| Node | Node | Distance |
|  |  |  |
| 1 | 2 | 110 |
| 1 | 3 | 150 |
| 1 | 4 | 190 |
| 2 | 3 | 215 |
| 2 | 4 | 275 |
| 3 | 4 | 310 |

|  |  |
| --- | --- |
| **1** | **Hotel** |
| **2** | **Tennis courts** |
| **3** | **Pool** |
| **4** | **Spa** |

**Problem 4:**

Formulate an *integer* linear programming problem from the information provided below:

We want to buy two types of machines (M1 and M2); use variables X1 and X2 to denote number of machines of two types.

Objective function: Marginal daily profitability per unit of M1 and M2 is 100 dollars and 150 dollars and you want to maximize your total daily profitability.

Resource and production constraints:

* M1 costs 15000 dollars per unit and M2 costs 4000 dollars. Your budget: 140,000 dollars.
* Space available: 200 square feet. M1 needs 15 square feet per unit and M2 needs 25 square feet per unit.
* Number of machines of type M1 must be at least twice as many as type M2.

List integer and/or other constraints at the end.

**Problem 5:**

Table 1 shows the payoff values for 3 alternative investment options and 3 events. Which alternative would you select if you use Mini-max regret approach?

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1: Payoff table | | | |  | Table 2: Regret table | | | | |
|  | Rates up | Rates static | Rates down |  |  | Rates up | Rates static | Rates down |  |
| Stocks | **-4** | **4** | **12** |  | Stocks |  |  |  |  |
| Bonds | **-2** | **3** | **8** |  | Bonds |  |  |  |  |
| Money M | **3** | **2** | **1** |  | Money M. |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

**Problem 6:**

|  |  |  |  |
| --- | --- | --- | --- |
| **Table 1: Payoff** | **Events** | | |
| **E1** | **E2** | **E3** |
| **A1** | 250 | 80 | 30 |
| **A2** | 150 | 140 | 130 |
| **Prob.** | 0.6 | 0.1 | 0.3 |

Table 1 below shows the payoff values with two alternatives A1 and A2. Each alternative has three chance events (E1, E2 and E3) with probabilities shown.

Draw a tree diagram, calculate expected value (EV) for each alternative and select the preferred alternative under risk-neutrality.

**Problem 7:**

A company has to decide whether to expand or not. The expansion cost is $2.1 million. If the economy improves substantially (20% probability) the (projected) revenues with expansion will be $6.0 million. If the economy turns worse (30% probability) the revenues will be $2.1 million. With economy remaining roughly the same (50% probability), the revenues will be 4.2 million. Without expansion, the corresponding revenues will be $3.0 million, $1.2 million and 1.9 million.

1. Draw a decision tree and calculate the expected value of each alternative.
2. What decision would you recommend to a risk-neutral decision-maker? Do not forget to take into account the cost of expansion.

**Problem 8 (from 2018 final exam):**

The associate dean at the Karey Business School needs to allocate instructors to the courses offered in the next semester. There are three instructors, each of whom will be teaching exactly *one* module. The past evaluation scores of these instructors are as follows:

|  |  |
| --- | --- |
| *Prof. Ford* | 60 |
| *Prof. Johnson* | 80 |
| *Prof. Hoover* | 50 |

There are 3 modules, each of which has a different number of students, as shown below:

|  |  |
| --- | --- |
| *Module 1: Leadership (qualitative)* | 35 students |
| *Module 2: Data analysis (quantitative)* | 45 students |
| *Module 3: Financial modeling (quantitative)* | 50 students |

1. (5 points) The associate dean wants to build a model for assigning the instructors to the modules. Define the decision variables for this model.
2. (10 points) Formulate a linear program with the objective to maximize total satisfaction (the sum of satisfaction of all students). You can assume that past evaluation scores reflect the satisfaction with each instructor, and that each instructor is capable of teaching each module.

|  |  |  |
| --- | --- | --- |
|  | *Qualitative courses* | *Quantitative courses* |
| *Prof. Ford* | *55* | *75* |
| *Prof. Johnson* | *80* | *80* |
| *Prof. Hoover* | *70* | *60* |

1. (10 points) It has been noticed that the instructors received different evaluation scores in the different courses they have taught. The scores are as follows:

Specifically, the dean wants to use the evaluation scores for qualitative courses to predict student satisfaction in module 1, and the evaluation scores for quantitative courses to predict student satisfaction in modules 2 and 3. Formulate a new model using these separate evaluation scores.

**Problem 9 (from 2019 final exam)**

You have built a fast food business and are now considering selling it. There are several potential acquirers, and you plan to determine the sale price in a sealed bid auction. Under the rules of the auction, the price would be the highest bid.

Due to their limited information the acquirers may underestimate or overestimate what the business is worth. You have received indication that McDonald’s valuation of your business is uniformly distributed between $100 Million and $120 Million, and that Burger King’s valuation of your business is uniformly distributed between $100 Million and $150 Million. You believe that both companies will bid their valuation during the auction.

In preparation for the auction you want to use simulation to see how much money you can make. You have generated the following random numbers from the [0,1] uniform distribution (each of the ten numbers has been generated independently):

|  |  |  |
| --- | --- | --- |
|  | McDonald’s | Burger King |
| Trial 1 | 0.46 | 0.06 |
| Trial 2 | 0.56 | 0.68 |
| Trial 3 | 0.97 | 0.33 |
| Trial 4 | 0.74 | 0.73 |
| Trial 5 | 0.15 | 0.12 |

1. (2 points) How can you use these numbers to predict the bids and the final sale price? Describe the approach.
2. (4 points) Given the numbers in the table above, what sale price do you expect, on average?
3. (2 points) The bidders have an instinct about the Winner’s curse and they each plan to bid only 75% of their valuation (Winner’s curse is a phenomenon where the winner generally overpays during competitive bidding). How would that change your answer to (b)?
4. (2 points) BARM Analytica, a startup out of Washington DC, offers to identify an additional bidder whose valuation is equally likely to be 100, 120 or 140. How large a fee should you be willing to pay for this service assuming that each bidder bids their true valuation? Use simulation and the following uniform [0,1] random numbers: 0.11, 0.65, 0.89, 0.23, 0.53 to justify your answer.

**Problem 10 (from 2020 final exam)**

In their development of a new medical treatment, GSK plc would like to know how much profit it might expect to make. The development is currently in phase III clinical trials, awaiting the results of testing on 100 patients. The FDA will approve the treatment if it is effective in at least 85% of the patients. Based on similar trials conducted by other drug companies, GSK expects the number of patients in which the treatment will be effective to be distributed *binomially*, with parameters 100 and 0.88.

The chart below visualizes the Binomial(100, 0.88) distribution. Note that x in the chart is the number (between 0 and 100) of the subjects in which the treatment is effective, and P(X=x) on the y-Axis is the probability of that outcome.

Chart, histogram

Description automatically generated

If the treatment is approved, GSK will incur R&D expenses which will be uniformly distributed between $30 and $50 Million. If the treatment is not approved there are no expenses. If the treatment is approved, then GSK will obtain a market share that is normally distributed with a mean of 8% and a standard deviation of 2%. The size of the market is $1Bn.

Lastly, there is a 1-in-4 chance of a competitor entering the market, and if it does, GSK’s market share will be reduced by 25, 50 or 75 percent of the original market share, with equal probabilities.

1. If the FDA approves the treatment, and the market share is *exactly* 8%, and the competitor does not enter the market, what is GSK’s expected profit? Answer this question without using simulation. (4 points)

*Use simulation to answer the following questions. To generate Binomial random numbers in Excel use binom.inv(X,Y,Z), where X and Y are the parameters of the binomial distribution and Z is the uniform(0,1) random number. Include the answers into your Word document. If your simulation approach is not clear from the Excel file, you can also describe your approach in words.*

1. What is GSK’s expected profit today (before FDA’s decision is known)? (6 points)
2. Based on your simulation, what is the probability that GSK makes more no or negative profit? (2 points)

**Solutions**

**Problem 1:**

**GAP words**

**O:** Maximize total profit

**D:** Numbers of the two products, A and B, produced

**C:** wood, plastic and steel limits

**GAP math**

**D:** A = number of product A produced, B = number of product B produced

**O:** Max 4 \* A + 6 \* Y

**C:**

Wood: A + 3 B <= 240

Plastic: 3A + 4B <= 360

Steel: 2A + B <= 180

A, B Nonnegative (Note that they need not be integer – it was not specified what type of products they are)

**Problem 2:**

**GAP words**

**O:** Maximize total profit

**D:** Numbers of the chairs and desks, produced

**C:** wood, labour and ratio limits

**GAP math**

**D:** D = number of desks produced, C = number of chairs produced

**O:** Max 40 \* D + 22 \* C

**C:**

Wood: 5D + 3C <= 2200

Labour: 3D + 1.4C <= 1150

Ratio: 3D – C <= 0

D, C Nonnegative and integer

**Problem 3:**

**GAP words:**

**O:** Minimize total distance

**D:** whether we connect two nodes or not

**C:** all locations must be connected

**GAP math:**

**D**: R12 (route from node 1 to node 2), R13, R14, R23, R24, R34

**O:** Minimize 110\*R12 + 150\*R13 + 190\*R14 + 215\*R23 + 275\*R24 + 310\*R34

**C:**

Each node must be connected to another node:

R12 + R13 + R14 >= 1

R12 + R23 + R24 >= 1

R13 + R23 + R34 >= 1

R14 + R24 + R34 >= 1

All nodes are connected:

R12 + R13 + R14 + R23 + R24 + R34 >= 3

(We need this additional constraint, otherwise R12 = R34 = 1 would satisfy all constraints but leave some of the nodes disconnected)

All variables are binary

**Problem 4:**

**GAP words:**

**O:** maximize total daily profitability

**D**: numbers of machines produces

**C:** budget, space limit and production ratio

**GAP Math:**

**D**: X1 = number of type M1 machines produced, X2 = number of type M2 machines produced

**O:** Maximize 100 \* X1 + 150 \* X2

**C:**

Budget: 15000X1 + 4000X2 <= 140000

Space: 15X1 + 25X2 <= 200

Production ratio: X1 – 2X2 >= 0

X1 and X2 are integer

**Problem 5:**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1: Payoff table | | | |  | Table 2: Regret table | | | | |
|  | Rates up | Rates static | Rates down |  |  | Rates up | Rates static | Rates down | **Maximum regret** |
| Stocks | **-4** | **4** | **12** |  | Stocks | **7** | **0** | **0** | **7** |
| Bonds | **-2** | **3** | **8** |  | Bonds | **5** | **1** | **4** | **5** |
| Money M. | **3** | **2** | **1** |  | Money M. | **0** | **2** | **11** | **11** |
| **Maximum** | **3** | **4** | **12** |  | **Select bonds (with minimax regret)** | | | | **5** |

**Problem 6:**

A1

60%

10%

30%

60%

10%

30%

250

80

30

150

140

130

A2

**Answers:**

a)

EV for A1 = 250\*0.6 + 80\*0.1 + 30\*0.3 = 167

EV for A2 = 150\*0.6 + 140\*0.1 + 130\*0.3 = 143

Select A1

**Problem 7**:

EV = 0.2 \* 3.9 + 0.3 \* 0 + 0.5 \* 2.1 = $1.83m

30%, 2

50%, 4.2

20%, 6

Expand

Do not expand

30%, 1.2

50%, 1.9

20%, 3

6.0 – 2.1 = 3.9

2.1 – 2.1 = 0

4.2 – 2.1 = 2.1

3.0

1.2

1.9

EV = 0.2 \* 3.0 + 0.3 \* (1.2) + 0.5 \* 1.9 = $1.91m

a)

1. Decision: EV(Expand) < EV (Do not expand) 🡪 Do not expand

**Problem 8**

**GAP words:**

**O:** Maximize total satisfaction

**D:** Module assignments for professors

**C:** Each professor can only teach one module

**GAP Math;**

**D:** AFL (assign Prof. Ford to Leadership Module) = 1 if Prof. Ford is assigned to Leadership Module, 0 otherwise, AFD, AFF, AJL, AJD, AJF, AHL, AHD, AHF

**O:** Maximize 60 \* 35 \* AFL + 60 \* 45 \* AFD + 60 \* 50 \* AFF + 80 \* 35 \* AJL + 80 \* 45 \* AJD + 80 \* 50 \* AJF + 50 \* 35 \* AHL + 50 \* 45 \* AHD + 50 \* 50 \* AHF

**C:**

Each instructor can only teach one module:

AFL + AFD + AFF = 1

AJL + AJD + AJF = 1

AHL + AHD + AHF = 1

Each module can be taught by only one instructor:

AFL + AJL + AHL = 1

AFD + AJD + AHD = 1

AFF + AJF + AHF = 1

All DVs are binary

**Answers**

a) See GAP Math part

b) See GAP Math part

c) We need to change the objective function:

Maximize: 55 \* 35 \* AFL + 75 \* 45 \* AFD + 75 \* 50 \* AFF + 80 \* 35 \* AJL + 80 \* 45 \* AJD + 80 \* 50 \* AJF + 70 \* 35 \* AHL + 60 \* 45 \* AHD + 60 \* 50 \* AHF

**Problem 9:**

1. If we denote the random number for McDonalds by X\_M and the random number for Burger King by X\_B For each number, then proceed as follows:
   1. Calculate Bid\_M = X\_M\*(120-100)+100
   2. Calculate Bid\_B = X\_B\*(150-100)+100
   3. Calculate Max (Bid\_M, Bid\_B) to get final sale price
   4. Take the average of the sale prices over 5 trials.
2. See table below.

|  |  |  |
| --- | --- | --- |
| Bid\_M | Bid\_B | Sale price (Max of col. 1 and col. 2) |
| 109.2 | 103 | 109.2 |
| 111.2 | 134 | 134 |
| 119.4 | 116.5 | 119.4 |
| 114.8 | 136.5 | 136.5 |
| 103 | 106 | 106 |
|  | **Average** | **121.02** |

1. See table below (columns equal to part b times 0.75)

|  |  |  |
| --- | --- | --- |
| Bid\_M | Bid\_B | Sale price (Max of col. 1 and col. 2) |
| 81.9 | 77.25 | 81.9 |
| 83.4 | 100.5 | 100.5 |
| 89.55 | 87.375 | 89.55 |
| 86.1 | 102.375 | 102.375 |
| 77.25 | 79.5 | 79.5 |
|  | Average | 90.765 |

1. See table below (Bids equal to part b times 0.75)

|  |  |  |  |
| --- | --- | --- | --- |
| Bid\_M | Bid\_B | Bid\_BARM | Sale price (Max of col. 1 and col. 2) |
| 109.2 | 103 | 100 | 109.2 |
| 111.2 | 134 | 120 | 134 |
| 119.4 | 116.5 | 140 | 140 |
| 114.8 | 136.5 | 100 | 136.5 |
| 103 | 106 | 120 | 120 |
|  |  | Average | 127.94 |
|  |  | Fee | 127.94-121.02= 6.92 |

**Problem 10:**

1. Expected R&D Expenses: $40 million

Market share value: 8% \* 1,000 million = $80 million

Expected Profits: 80 – 40 = 40 million

1. Answer: average profit is $23-27 Million. See Excel file for details.
2. Answer: around 25-29%. See Excel file for details.